INDIAN SCHOOL DARSAIT

Class XII — ***Mathematics Worksheet***

(Chapter – 1: Relations & Functions)

CLASS WORK	
1.	Show that the function f: R → R defined by f(x)= 3 – 4x is one – one and onto (Bijective)
2.	Let A = R – {3}, B = R – {1}. Consider the function f: A → B defined by $f(x)=\frac{x-1}{x-3}$. Prove that f is one – one and onto.
3.	Let A = R – {3}, B = R - $\left\{\frac{2}{3}\right\}$. If f: A → B by f(x) = $\frac{2x-4}{3x-9}$, prove that f is a bijection
4.	Show that the function f: R → R defined by f(x)= $x^3 + x$ is a bijection
5.	Show that f: N → N given by f(n) = n – $(-1)^n$ for all n ∈ N is a bijection.
6.	Show that the function f: N → N given by $f(n)=\begin{cases}\frac{n+1}{2} & if\ \ n\ is\ odd\\ \frac{n}{2} & if\ \ n\ is\ even\end{cases}$ is many – one onto function.
7.	Show that f: N → N defined by f(x) = $x^2 + x + 1$ is one – one but not onto.
8.	Show that the signum function f: R → R given by $f(x)=\begin{cases}1\ if\ x>0\\ 0\ if\ x=0\\ -1\ if\ x<0\end{cases}$ is neither one – one nor onto.
9.	Find fog and gof if f, g: R → R given by i) f(x) = Sinx, g(x) = $4x^2$ ii) f(x)= x^2 , g(x) = 2x + 1
10.	If the function f : R → R given by f(x) = $x^2 + 2$ and g: R →R given by g(x) = $\frac{x}{x-1}$, then find fog and gof. Hence find fog (2) and gof (-3).
11.	If f be the greatest integer function and g be the modulus function. Find the value of $gof\left(\frac{-1}{3}\right)-fog\left(\frac{-1}{3}\right)$.
12.	Let A = $R-\left\{\frac{7}{5}\right\}$, B = $R-\left\{\frac{3}{5}\right\}$. If f : A → B defined by and g : B → A by $f(x)=\frac{3x+4}{5x-7}$ $g(x)=\frac{7x+4}{5x-3}$, then prove that gof = I_A and fog = I_B.
13.	If f,g : R →R be two functions defined by $f(x)=\|x\|+x$ and $g(x)=\|x\|-x$, ∀ x ∈R, then find fog and gof. Also find fog(-3), fog(5) and gof(-2)
14.	Consider f: {1,2,3} → {a, b, c} given by f(1) = a, f(2) = b, f(3) = c. Find f^{-1} . Show that $\left(f^{-1}\right)^{-1}=f$.

15.	If f : R → R be a function defined by f(x) = 3x – 7 , show that f is invertible. Find f^{-1}
16.	If f : R → R be a function defined by f(x) = 10x + 7 , show that f is invertible. Find f^{-1}
17.	If f is an invertible function defined by $f(x)=\frac{3x-2}{5}$, find f^{-1}
18.	Let f: $R-\left\{\frac{-4}{3}\right\}$→R be a function defined by $f(x)=\frac{4x}{3x+4}$. Prove that f: S → $R-\left\{\frac{-4}{3}\right\}$, where S is the range of f, is invertible. Also find the inverse of f.
19.	Find the inverse of the function f(x) = $\frac{a^x-a^{-x}}{a^x+a^{-x}}$
20.	Find the value of the parameter α for which the function f(x) = 1 + αx, α≠ 0, is the inverse of itself.
21.	Consider f: R_+ → [-5,∞) given by f(x) = 9x² + 6x – 5. Show that f is invertible with $f^{-1}(y)=\frac{\sqrt{y+6}-1}{3}$
22.	Let f: $[0,\infty)\to R$ be a function defined by f(x) = 9x² + 6x – 5. Prove that f is not invertible. Modify codomain of f to make it invertible and find its inverse.
23.	Let f: N → R be a function given by f(x) = 4x² + 12x +15. Show that f: N → S, where S is the range of f, is invertible. Also find the inverse of f and hence find $f^{-1}(31)$ and $f^{-1}(87)$
24.	Prove that f : R_+ → [-9,∞) given by f(x) = 5x² +6x – 9. Prove that f is invertible. Find the inverse of f.
25.	Prove that f : R_+ → [-5,∞) given by f(x) = 3x² +2x – 5. Prove that f is invertible. Find the inverse of f.
26.	Consider f : {1,2,3} → {a,b,c} and g : {a,b,c} → {apple, ball, cat} defined as f(1) =a, f(2) = b, f(3) = c g(a) = apple, g(b) = ball, g(c) = cat. Show that f, g and gof are invertible. Find out f^{-1}, g^{-1} and $(gof)^{-1}$. Also show that $(gof)^{-1} = f^{-1}og^{-1}$.
27.	Let A = {1,2,3,4} B = {3,5,7,9} and C = {7,23,47,79}. f; A → B, g: B → C defined by f(x) = 2x +1, g(x) = x² – 2. Express $(gof)^{-1}$ and $f^{-1}og^{-1}$ as set of ordered pairs and verify that $(gof)^{-1} = f^{-1}og^{-1}$
28.	If f : W→W defined as $f(x)=\begin{cases}x-1, & if\ x\ is\ odd\\ x+1, & if\ x\ is\ even\end{cases}$, show that f is invertible. Find the inverse of f, where W is the set of whole numbers.
29.	If the function f: R → R be defined by f(x) = 2x – 3, g: R → R by g(x) = x³ + 5. Find fog. Hence show that fog is invertible. Also find $(fog)^{-1}$ and $(fog)^{-1}(9)$
	HOME WORK
30.	Show that the function f: R → R defined by f(x)= ax + b is a bijection.

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31.	Let f: N∪{0} → N∪{0} defined by $f(n)=\begin{cases} n+1 \text{ if } n \text{ is even} \\ n-1 \text{ if } n \text{ is odd}\end{cases}$. Show that f is a bijection
32.	Show that f: {x∈R: −1 < x < 1 defined by $f(x)=\frac{x}{1+\lvert x\rvert}$, x ∈R is a one – one onto function
33.	Show that f : R → R defined by i) $f(x)=\frac{4x-3}{5}, x\in R$ ii) $f(x)=\frac{3x-1}{2}, x\in R$ is one – one and onto function.
34.	Show that f : [-1,1] → R, given by $f(x)=\frac{x}{x+2}$ is one – one. Find the inverse of f : [-1,1] → range of f.
35.	Show that the function f: R → R defined by f(x) = $2x^3 - 7$ is a bijection.
36.	Prove that the modulus function f(x) = |x| is neither one – one nor onto.
37.	If f,g : N → R be the function defined by $f(x)=\frac{2x-1}{2}$ and g(x) = x + 2, then find $gof\left(\frac{3}{2}\right)$.
38.	If f : R → R is given by $f(x)=\left(3-x^3\right)^{1/3}$, find fof(x).
39.	If f and g are two functions given by f = {(1,2), (3,5), (4,1)} , g = {(2,3), (5,1), (1,3)}, find fog.
40.	If the function $f:[1,\infty)\to[1,\infty)$ defined by $f(x)=2^{x(x-1)}$ is invertible, find $f^{-1}(x)$
41.	Let A = {-1,0,1,2} B = {-4, -2,0,2}. f,g : A → B be two functions defined by f(x) = $x^2 - x$, ∀ x∈A, $g(x)=2\left\lvert x-\frac{1}{2}\right\rvert-1$, ∀ x∈A. Are f and g equal? Justify your answer.
42.	Consider f: $R_+ \to [4,\infty)$ given by f(x) = $x^2 + 4$. Show that f is invertible with $f^{-1}(y)=\sqrt{y-4}$, where R_+ is the set of all non – negative real numbers.
43.	Let f: N → R be a function given by f(x) = $x^2 + 4x + 7$. Show that f: N → S, where S is the range of f, is invertible. Also find the inverse of f .
44.	If f: R → R be a function defined by f(x) = 4x + 3 , show that f is invertible. Find f^{-1}
	SELF STUDY
45.	Show that f: R → R defined by f(x) = x^2 is neither one – one nor onto.
46.	Write the number of one – one functions from {a, b, c} to itself.
47.	Show that an onto function f: {1, 2, 3} → {1, 2, 3} is always one – one
48.	Let A and B are any two sets. Show that f: A×B → B×A defined by f(a,b) = (b,a) is a bijective function.
49.	Show that the function f in A = R - $\left\{\frac{2}{3}\right\}$ defined by f(x) = $\frac{4x+3}{6x-4}$ is one – one and onto. Find f^{-1}.

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Class XII **Mathematics Worksheet**

(Chapter – 1: Relations & Functions)

50.	Let A = $\{x \in R, -1 \le x \le 1\}$ = B. Show that f: A → B given by f(x) = x|x| is a bijection.
51.	Let R_0 be the set of non – zero real numbers. Show that f : R → R given by f(x) = $\frac{1}{x}$ is one – one and onto.
52.	Show that f : R → R given by $f(x) = \frac{x}{1+x^2}$ ∀ x∈ R is neither one – one nor onto.
53.	Let f: R → R defined by $f(x) = \begin{cases} 2x, & x > 3 \\ x^2, & 1 < x \le 3 \\ 3x, & x \le 1 \end{cases}$, find f(-1) + f(2) + f(4)
54.	Let [0,1] → [0,1] be defined by $f(x) = \begin{cases} x, & if\ x\ is\ rational \\ 1-x, & if\ x\ is\ irrational \end{cases}$, find (fof)x
55.	If X and Y are two sets having 2 and 3 elements respectively, find the number of functions from X to Y.
56.	If f: R →R be the signum function given by $f(x) = \begin{cases} 1, & if\ x > 0 \\ 0, & if\ x = 0 \\ -1, & if\ x < 0 \end{cases}$ and g: R → R be the greatest integer function given by g(x) = [x], where [x] is the greatest integer less than or equal to x. Does gof and fog coincide in [0,1]?
57.	Let g(x)=1+x – [x] and $f(x) = \begin{cases} 1, & if\ x > 0 \\ 0, & if\ x = 0 \\ -1, & if\ x < 0 \end{cases}$, find fog(x).
58.	Find fog and gof if $i)\ f(x) = \|x\|,\quad g(x) = \|5x-2\| \quad ii)\ f(x) = 8x^3\ and\ g(x) = x^{1/3}$
59.	Let f: N → N, g: N → N and h: N → R defined as f(x) = 2x, g(y) = 3y + 4 and h(z) = Sinz ∀ x, y, z∈ N. Show that ho(gof) = (hog)of.
60.	Let f, g, h be functions from R to R, show that (f + g)oh = f o h + g o h
61.	Consider $f, g : \left[0, \frac{\pi}{2}\right] \to R$ given by f(x) = sinx and g(x) = Cosx. Show that f and g are one – one but f + g is not one – one.
62.	If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x+x^3}{1+3x}$, then find the value of $(gof)\left(\frac{e-1}{e+1}\right)$
63.	If $f(x) = \frac{4x+3}{6x-4}, x \ne \frac{2}{3}$, show that fof(x) =x ∀ $x \ne \frac{2}{3}$. What is the inverse of f ?
64.	If $f(x) = \frac{x-1}{x+1}$, (x ≠ −1, 1), show that fof^{-1} is an identity function.