

INDIAN SCHOOL DARSAIT

Class XII

Mathematics Worksheet

Worksheet # 4 Inverse Trigonometric Functions (Chapter – 2: Inverse Trigonometric Functions)

CLASS WORK

1.	Find the value of: i) $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ ii) $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) + \operatorname{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right)$ iii) $\sin^{-1}\left(-\frac{1}{2}\right) + 2\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ iv) $\sin^{-1}\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$ v) $\cos^{-1}\left(\frac{3}{5}\cos x + \frac{4}{5}\sin x\right)$
2.	Evaluate: i) $\sin^{-1}\sin\left(\frac{\pi}{3}\right)$ ii) $\cos^{-1}\cos\left(\frac{2\pi}{3}\right)$ iii) $\tan^{-1}\tan\left(\frac{3\pi}{4}\right)$ iv) $\cos^{-1}\cos\left(\frac{7\pi}{6}\right)$ v) $\sin^{-1}\sin\left(\frac{2\pi}{3}\right) + \cos^{-1}\cos\left(\frac{2\pi}{3}\right)$ vi) $\cos^{-1}\cos\left(\frac{13\pi}{6}\right) + \tan^{-1}\tan\left(\frac{5\pi}{6}\right)$
3.	Evaluate: i) $\sin\left[2\cos^{-1}\left(\cot\left(2\tan^{-1}x\right)\right)\right] = 0$ ii) $\cos\left(\sec^{-1}x + \operatorname{cosec}^{-1}x\right)$
4.	Find the value of: i) $\sin\left(\tan^{-1}x\right)$ ii) $\cos\left(\cot^{-1}x\right)$
5.	Simplify: $\tan^{-1}\frac{x}{y} - \tan^{-1}\frac{x-y}{x+y}$
6.	Simplify: i) $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$, $-\pi < x < \pi$ ii) $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$, $-\pi/2 < x < \pi/2$ iii) $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, $-\pi/2 < x < \pi/2$ iv) $\tan^{-1}\sqrt{\frac{a-x}{a+x}}$, $-a < x < a$
7.	Simplify: i) $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$, $-\pi/4 < x < \pi/4$, ii) $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$ iii) $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right)$ iv) $\tan^{-1}\left[\frac{\sqrt{1+x^2}+1}{x}\right]$ v) $\tan^{-1}\left[\sqrt{1+x^2} - x\right]$ vi) $\sin\left[2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right]$
Prove the following: -	
8.	i) $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{77}{85}$ ii) $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$
9.	i) $\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \pi$ ii) $\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65} = \frac{\pi}{2}$
10.	$\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$
11.	i) $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$ ii) $\tan^{-1}2 + \tan^{-1}3 = \frac{3\pi}{4}$
12.	i) $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$ ii) $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\cos^{-1}\frac{3}{5}$
13.	i) $2\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \tan^{-1}\frac{4}{7}$ ii) $2\tan^{-1}\frac{1}{3} + \sin^{-1}\frac{4}{5} = \frac{\pi}{2}$
14.	i) $2\tan^{-1}\frac{1}{5} + \sec^{-1}\frac{5\sqrt{2}}{7} + 2\tan^{-1}\frac{1}{8} = \frac{\pi}{4}$ ii) $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 = \cot^{-1}3$

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(Chapter – 2: Inverse Trigonometric Functions)**

15.	$\tan^{-1} \left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right] = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2$
16.	i) $\tan^{-1} \left[\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} \right] = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x$ ii) $\cot^{-1} \left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} \right] = \frac{x}{2}$
Solve the following: -	
17.	If $\tan^{-1} x = \frac{\pi}{10}$, $x \in \mathbb{R}$, find the value $\cot^{-1} x$
18.	If $3 \tan^{-1} x + \cot^{-1} x = \pi$, find the value of x
19.	i) $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$ ii) $\tan^{-1} \left(\frac{x-1}{x-2} \right) + \tan^{-1} \left(\frac{x+1}{x+2} \right) = \frac{\pi}{4}$ iii) $\tan^{-1} \left(\frac{x-1}{x+1} \right) + \tan^{-1} \left(\frac{2x-1}{2x+1} \right) = \tan^{-1} \left(\frac{23}{36} \right)$ iv) $\cot^{-1} \frac{1}{x+1} + \cot^{-1} \frac{1}{x-1} = \tan^{-1} 3x - \tan^{-1} x$
20.	i) $\tan^{-1} \left(\frac{1-x}{1+x} \right) = \frac{1}{2} \tan^{-1} x$, $x > 0$ ii) $2 \tan^{-1} \cos x = \tan^{-1} (2 \cos ecx)$
21.	i) $\sin^{-1} (1-x) - 2 \sin^{-1} x = \frac{\pi}{2}$ ii) $\sin^{-1} x + \sin^{-1} (1-x) = \cos^{-1} x$
22.	i) $\sin^{-1} 6x + \sin^{-1} 6\sqrt{3}x = -\frac{\pi}{2}$ ii) $\tan^{-1} \left(\frac{1-x}{1+x} \right) - \frac{1}{2} \tan^{-1} x = 0$, $x > 0$ iii) $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$
23.	If $\tan^{-1} x + \tan^{-1} y = \frac{4\pi}{5}$, find the value of $\cot^{-1} x + \cot^{-1} y$
24.	Evaluate: $\tan^{-1} \left[\frac{3 \sin 2\alpha}{5 + 3 \cos 2\alpha} \right] + \tan^{-1} \left(\frac{1}{4} \tan \alpha \right)$
25.	Prove that $\tan \left[\frac{\pi}{4} + \frac{1}{2} \cos^{-1} \left(\frac{a}{b} \right) \right] + \tan \left[\frac{\pi}{4} - \frac{1}{2} \cos^{-1} \left(\frac{a}{b} \right) \right] = \frac{2b}{a}$
26.	If $a > b > c > 0$, prove that $\cot^{-1} \left(\frac{ab+1}{a-b} \right) + \cot^{-1} \left(\frac{bc+1}{b-c} \right) + \cot^{-1} \left(\frac{ca+1}{c-a} \right) = \pi$
27.	If $\cos^{-1} \left(\frac{x}{a} \right) + \cos^{-1} \left(\frac{y}{b} \right) = \alpha$, prove that $\frac{x^2}{a^2} - \frac{2xy \cos \alpha}{ab} + \frac{y^2}{b^2} = \sin^2 \alpha$
28.	If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$
29.	If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, prove that $xy + yz + zx = 1$
30.	Prove that $2 \tan^{-1} \left[\frac{\sqrt{a-b} \tan \left(\frac{x}{2} \right)}{\sqrt{a+b}} \right] = \cos^{-1} \left[\frac{b+a \cos x}{a+b \cos x} \right]$
31.	Prove that $\frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \left(\frac{1}{3} \right) = \frac{9}{4} \sin^{-1} \left(\frac{2\sqrt{2}}{3} \right)$
32.	Prove that $\tan \frac{1}{2} \left[\sin^{-1} \left(\frac{2x}{1+x^2} \right) + \cos^{-1} \left(\frac{1-y^2}{1+y^2} \right) \right] = \frac{x+y}{1-xy}$

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33.	Find the greatest and least values of $(\sin^{-1} x)^2 + (\cos^{-1} x)^2$
34.	If $\alpha = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^{n-1}}$ and $\beta = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots + \frac{1}{3^{n-1}}$ are two geometric series, find the value of $\sec^{-1} \alpha + \sec^{-1} \beta$
35.	If a_1, a_2, a_3, \dots are in A.P with common difference d, evaluate $\tan \left[\tan^{-1} \left(\frac{d}{1+a_1 a_2} \right) + \tan^{-1} \left(\frac{d}{1+a_2 a_3} \right) + \dots + \tan^{-1} \left(\frac{d}{1+a_{n-1} a_n} \right) \right]$

HOME WORK

36.	Evaluate : i) $\sin^{-1} \sin \left(\frac{3\pi}{5} \right) + \cos^{-1} \cos \left(\frac{13\pi}{6} \right)$ ii) $\tan^{-1} \tan \left(\frac{7\pi}{4} \right) + \operatorname{cosec}^{-1} \operatorname{cosec} \left(\frac{13\pi}{4} \right)$
37.	Find the value of : i) $\cos^{-1} \left(\frac{1}{2} \right) + 2 \sin^{-1} \left(\frac{1}{2} \right)$ ii) $\sin^{-1} \left(\frac{1}{2} \right) - 2 \sin^{-1} \left(\frac{1}{\sqrt{2}} \right)$ iii) $\tan^{-1} (-1) + \cos^{-1} \left(-\frac{1}{\sqrt{2}} \right)$ iv) $\sin^{-1} \left(-\frac{\sqrt{3}}{2} \right) + \cos^{-1} \left(\frac{\sqrt{3}}{2} \right)$
38.	Find the value of : i) $\sin \left(\frac{1}{2} \cos^{-1} \frac{4}{5} \right)$ ii) $\sin (\tan^{-1} x + \cot^{-1} x)$
39.	Simplify: i) $\cos^{-1} \left(\frac{x}{\sqrt{a^2 + x^2}} \right)$ ii) $\sin^{-1} \left(\frac{\sin x + \cos x}{\sqrt{2}} \right), -\pi/4 < x < \pi/4$ iii) $\tan^{-1} \left(\frac{x}{\sqrt{a^2 - x^2}} \right), -a < x < a$ iv) $\sin^{-1} \left(\frac{x}{\sqrt{a^2 + x^2}} \right) \tan^{-1} [x + \sqrt{1+x^2}]$ v) $\tan^{-1} \left[\frac{\sqrt{1+x^2} + 1}{x} \right]$ vi) $\cot^{-1} \left[\frac{a}{\sqrt{a^2 - x^2}} \right]$ vii) $\tan^{-1} \left(\frac{2\sqrt{x}}{1-x} \right)$
40.	Simplify: i) $\cos^{-1} \left(\frac{\sin x + \cos x}{\sqrt{2}} \right), \pi/4 < x < 5\pi/4$ ii) $\tan^{-1} \left(\frac{a+bx}{a-bx} \right)$

Prove the following: -

41.	i) $\cos^{-1} \frac{12}{13} + \cos^{-1} \frac{4}{5} = \cos^{-1} \frac{33}{65}$ ii) $\sin^{-1} \frac{3}{5} - \sin^{-1} \frac{8}{17} = \cos^{-1} \frac{84}{85}$
42.	i) $\tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{13} = \tan^{-1} \frac{2}{9}$ ii) $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$
43.	$\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = 2 \left(\tan^{-1} 1 + \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} \right)$
44.	$\tan^{-1} \left[\frac{\sqrt{1+x^3} - \sqrt{1-x^3}}{\sqrt{1+x^3} + \sqrt{1-x^3}} \right] = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x^3$
45.	$\tan^{-1} \left[\frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}} \right] = \frac{\pi}{4} + \frac{x}{2}$

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46.	i) $\tan^{-1}\left(\frac{x-2}{x-4}\right) + \tan^{-1}\left(\frac{x+2}{x+4}\right) = \frac{\pi}{4}$ ii) $\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = \tan^{-1}(-7)$
47.	i) $\tan^{-1}\frac{1}{2x+1} + \tan^{-1}\frac{1}{4x+1} = \tan^{-1}\frac{2}{x^2}$
48.	$\sin^{-1}3x + \sin^{-1}3\sqrt{3}x = -\frac{\pi}{2}$
49.	i) $\tan^{-1}(x-1) + \tan^{-1}(x+1) = \tan^{-1}\frac{8}{31}$ ii) $\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}3x$
50.	Prove that $\cos^{-1}x = 2\sin^{-1}\sqrt{\frac{1-x}{2}} = 2\cos^{-1}\sqrt{\frac{1+x}{2}}$
51.	If $\cos^{-1}\left(\frac{x}{2}\right) + \cos^{-1}\left(\frac{y}{3}\right) = \alpha$, prove that $9x^2 - 12xy\cos\alpha + 4y^2 = 36\sin^2\alpha$
52.	If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, prove that $x + y + z = xyz$
53.	Prove that $\cos^{-1}x + \cos^{-1}\left[\frac{x + \sqrt{3-3x^2}}{2}\right] = \frac{\pi}{3}$
54.	If $\tan^{-1}\left(\frac{\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}}\right) = \frac{\pi}{4} - \frac{\alpha}{2}$, show that $x = \sin^2\alpha$
55.	If $\sin^{-1}\frac{2a}{1+a^2} + \sin^{-1}\frac{2b}{1+b^2} = 2\tan^{-1}x$, prove that $x = \frac{a+b}{1-ab}$
56.	Prove that $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$
57.	If $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$, find the value of $\cos^{-1}x + \cos^{-1}y$
SELF STUDY	
58.	Evaluate i) $\cos^{-1}\left(\frac{3}{5}\cos x + \frac{4}{5}\sin x\right)$ ii) $\cot(\tan^{-1}a + \cot^{-1}a)$
59.	Evaluate : i) $\sin^{-1}\left(\frac{5}{13}\cos x + \frac{12}{13}\sin x\right)$ ii) $\sin(\tan^{-1}x + \cot^{-1}x)$ iii) $\operatorname{cosec}\left(\cos^{-1}\left(-\frac{12}{13}\right)\right)$
60.	Simplify : $\sin^{-1}\left(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right)$
61.	Simplify : i) $\tan^{-1}\left[\frac{x}{a + \sqrt{a^2 - x^2}}\right]$ ii) $\sin^{-1}\left[\frac{x + \sqrt{1-x^2}}{\sqrt{2}}\right]$ iii) $\sin^{-1}\left[\frac{\sqrt{1+x} + \sqrt{1-x}}{2}\right]$
Prove the following:-	
62.	$\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = \frac{\pi}{4}$
Solve the following:-	
63.	i) $\tan^{-1}\frac{x}{2} + \tan^{-1}\frac{x}{3} = \frac{\pi}{4}$, $\sqrt{6} > x > 0$ ii) $2\tan^{-1}\sin x = \tan^{-1}(\sec x)$, $x \neq \frac{\pi}{2}$

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64.	$\tan^{-1}(x+2) + \tan^{-1}(x-2) = \tan^{-1} \frac{8}{79}$
65.	$\tan^{-1} \frac{1}{4} + 2 \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{6} + \tan^{-1} \frac{1}{x} = \frac{\pi}{4}$
66.	$\tan(\cos^{-1} x) = \sin\left(\cot^{-1} \frac{1}{2}\right)$
67.	$\cos(\tan^{-1} x) = \sin\left(\cot^{-1} \frac{3}{4}\right)$
68.	$\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{\pi}{4} + \tan^{-1} x$
69.	$\tan^{-1} x - \cot^{-1} x = \tan^{-1} \frac{1}{\sqrt{3}}$
70.	$\sin\left(\sin^{-1} \frac{1}{5} + \cos^{-1} x\right) = 1$
71.	Prove that $\tan^{-1} x + \tan^{-1} \frac{2x}{1-x^2} = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$
72.	Prove that $\tan^{-1}\left(\frac{2ab}{a^2-b^2}\right) + \tan^{-1}\left(\frac{2xy}{x^2-y^2}\right) = \tan^{-1}\left(\frac{2\alpha\beta}{\alpha^2-\beta^2}\right)$ where $\alpha = ax-by$ and $\beta = ay+bx$
73.	For a, b, x, y > 0, prove that $\frac{2}{3} \tan^{-1}\left(\frac{3ab^2-a^3}{b^3-3a^2b}\right) + \frac{2}{3} \tan^{-1}\left(\frac{3xy^2-x^3}{y^3-3x^2y}\right) = \tan^{-1}\left(\frac{2\alpha\beta}{\alpha^2-\beta^2}\right)$ where $\alpha = by-ax$ $\beta = bx+ay$
74.	Prove that $\cos^{-1}\left[\frac{\cos \alpha + \cos \beta}{1 + \cos \alpha \cos \beta}\right] = 2 \tan^{-1}\left[\tan\left(\frac{\alpha}{2}\right) \tan\left(\frac{\beta}{2}\right)\right]$
75.	If $y = \cot^{-1} \sqrt{\cos x} - \tan^{-1} \sqrt{\cos x}$, prove that $\sin y = \tan^2\left(\frac{x}{2}\right)$
76.	Show that $2 \tan^{-1}\left[\tan\left(\frac{\alpha}{2}\right) \tan\left(\frac{\pi}{4} - \frac{\beta}{2}\right)\right] = \tan^{-1}\left[\frac{\sin \alpha \cos \beta}{\cos \alpha + \sin \beta}\right]$
77.	Prove that $\sum_{m=1}^n \tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right) = \tan^{-1}\left(\frac{n^2+n}{n^2+n+2}\right)$
78.	Find the value of $\sum_{x=0}^n \tan^{-1}\left(\frac{1}{1+x+x^2}\right)$
79.	If $\cos^{-1} \alpha + \cos^{-1} \beta + \cos^{-1} \gamma = 3\pi$, find the value of $\alpha(\beta+\gamma) + \beta(\gamma+\alpha) + \gamma(\alpha+\beta)$
80.	Prove that $\tan^{-1}\left(\frac{yz}{xr}\right) + \tan^{-1}\left(\frac{xz}{yr}\right) + \tan^{-1}\left(\frac{xy}{zr}\right) = \frac{\pi}{2}$, where $x^2 + y^2 + z^2 = r^2$
81.	If x, y and z $\in [-1,1]$ such that $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, find the value of $x^{2006} + y^{2007} + z^{2008} + \frac{9}{x^{2006} + y^{2007} + z^{2008}}$